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PROBLEMS WITH DUALITY IN N=2 SUPER–YANG–MILLS THEORY

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Actual calculations of monopole and dyon spectra have previously been performed in N=4 SYM and in N=2 SYM with gauge group SU(2), and are in total agreement with duality conjectures for the finite theories. These calculations are extended to N=2 SYM with higher rank gauge groups, and it turns out that the SU(2) model with four fundamental hypermultiplet is an exception in that its soliton spectrum supports duality. This may be an indication that the other perturbatively finite N=2 theories have non-perturbative contributions to the β-function. This talk contains a short summary of recent results [1].

The last years have seen big progress in the understanding of non-perturbative aspects of quantum field theory and string theory. In supersymmetric enough theories, some non-perturbative properties may actually be calculated exactly [2,3,4,5,6]. Intimately connected with the new techniques developed is the somewhat older concept of duality [7,8]. Finite quantum field theories may, and seem to, exhibit a duality symmetry, which is the realization and extension of the old puzzling symmetry between electricity and magnetism in Maxwell theory. $\mathbb{Z}_2$ duality transforms the coupling constant of the theory to its inverse, and exchanges (non-abelian) electric and magnetic charges. When the theta angle is included in the complex coupling constant, the group of duality transformations is $Sl(2;\mathbb{Z})$, acting on the vector of electric and magnetic charge, and projectively on the coupling constant. Then also dyons, states carrying both electric and magnetic charge, are involved. The relevant magnetically charged states in a model containing Yang–Mills and a Higgs field are the magnetic (multi-)monopoles and dyons. If a theory is finite, and the coupling constant does not run, the transformations can make sense, and there is the possibility that duality is an exact symmetry of the theory. Then there is no fundamental difference between elementary and solitonic excitations, just that we need to chose one element in the duality group, *i.e.* a set of "elementary" excitations, in order to formulate the theory. A geometric understanding of this kind of

duality, in the context of string theory called S-duality, is still lacking, and is probably needed for our understanding of what string theory "really is".

There is an infinity of perturbatively finite quantum field theories. Of special interest among these are supersymmetric Yang–Mills theories with at least two supersymmetries, *i.e.* $N=2,4$. All $N=4$ super-Yang–Mills theories are perturbatively finite [9,10,11,12,13,14], and this property has been shown to hold also non-perturbatively [15]. The models with $N=2$ are perturbatively finite if the matter content is appropriate, *e.g.* the $SU(N_c)$ models with $N_f=2N_c$ hypermultiplets in the fundamental representation. It is of course important to decide whether these theories are non-perturbatively finite.

The questions of finiteness and duality are closely related. A non-running coupling constant is required for duality symmetry, and conversely, if there is a number of excitations that become massless as the coupling constant goes to some specific value in a finite theory, these should play the role of elementary excitations in a perturbation expansion of a finite theory. Examining the spectrum of solitons in a theory may therefore give information both about duality and finiteness.

The procedure for finding (part of) the spectra of monopoles and dyons in the theories at hand is well established [16,17,18,19,20,21,22]. Starting from a classical solution that is BPS-saturated, *i.e.* has a certain relation between mass and magnetic charge and breaks half of the supersymmetry, one makes a low-energy approximation. Here, the $N=2$ supersymmetry is crucial, in that the electric and magnetic charges enter the central charges in the supersymmetry algebra. BPS-saturated states come in short multiplets, with half the size of a full multiplet (this is identical to the difference between massless and massive multiplets, only that "$m=0$" is shifted in the presence of the central extensions). The number of states in a multiplet should not be quantum corrected and the BPS property is therefore valid also outside the low-energy approximation [23]. Due to the presence of zero-modes, bosonic and fermionic, and the presence of a mass-gap, the low-energy approximation gets only a finite number of degrees of freedom, namely the moduli parameters of the relevant topological sector (specified by magnetic charge) and fermionic zero-modes. The task of finding BPS states is, roughly speaking, reduced to that of finding ground states of a supersymmetric quantum mechanics model, which amounts to a cohomology problem. The riemannian geometry of the monopole moduli spaces and the geometry of the index bundles of fermionic zero-modes over these moduli spaces are essentially determined from the form of the kinetic terms in the field theory lagrangian, and the geometric considerations can be pushed quite far on a purely formal basis. What restricts the actual calculation

of ground states, is that it, at least this far, requires concrete knowledge of the moduli spaces. Only at the lowest values of the magnetic charge is the metric structure known explicitely, which restricts the calculations to these cases (note, however, the method of [17], which seems to circumvent this statement).

These calculations have been performed for the $N=4$ theories [16,17,21, 22], where the results are in perfect agreement with the predictions from duality. Also for the $N=2$ theory with gauge group $SU(2)$ and four hypermultiplets in the fundamental representation, duality (as predicted in [3]), is in agreement with the obtained spectrum [20,19]. We wanted to continue this investigation to include also the other perturbatively finite $N=2$ theories. The expectation was to find spectra that would lend themselves to a direct interpretation in terms of a dual finite theory. For a number of reasons, this did not happen.

It is quite easy to point at at least two problems with duality in $N=2$ theories with gauge groups of rank higher that one. The electric charges of the elementary excitation lie on the weight lattice of the gauge group, and the magnetic charges on the dual to this lattice (more precisely, on the dual to the weight lattice of the universal cover of the group), the coroot lattice, which for simply laced groups coincide with the root lattice. These two lattices are in general different, also modulo a rescaling (for $SU(4)$, they are the *fcc* and *bcc* lattices). Note that this only is an argument against $\mathbb{Z}_2$ symmetry of electric–magnetic interchange in the cases where the lattices differ, but in a finite theory this would also mean that one is not expecting to find any BPS-saturated states with purely magnetic charge, which we actually do. Also, there is a serious problem already with the classical moduli spaces, namely that only some sectors of the coroot lattice are allowed as magnetic charges. These are the coroots that in a suitable basis are sums of the simple coroots with only positive or only negative coefficients. The other coroots would correspond to superpositions of monopoles and anti-monopoles, *i.e.* of selfdual and anti-selfdual field configarations. Such a configuaration can not be BPS-saturated. In the $N=4$ case, where only the trivial conjugacy class of the weight lattice is populated with elementary excitations, this presents no problems. All the magnetic charges align with the magnetic ones, and the forbidden sectors are avoided. For $N=2$ theories with gauge groups of rank higher than one this is a problem.

Of course, indirect arguments like these, how convincing they might sound, can turn out to be wrong or irrelevant for reasons difficult to predict. Therefore, it is important to perform the actual calcualtion of the spectra in order to examine if there is some obvious sign of duality in some parts of them. We did not find any such signs. Some of the spectra for gauge group $SU(3)$ are

presented diagrammatically in [1].

What are the interpretations of these results? One must bear in mind that the non-perturbative finiteness of all these $N=2$ models has not been proven, and the results we have obtained would become very natural if there were instanton contributions to the β-function (with the exception of the finite $SU(2)$ theory). We do not think our evidence is strong enough for a claim of this type, but we feel that the possibility must be considered, even if it is against the common lore. Presumably, if the perturbatively finite models discussed here actually exhibit non-perturbative divergencies, they should manifest themselves as diverging sums over instanton number, since the contribution from a sector with given instanton number should be perturbatively finite, and the integrals over instanton moduli converge in the presence of a Higgs expectation value.

For concrete calculations, formulas and presentation of the spectra, I refer to [1], which also contains a more exhaustive list of references.

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